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## Job search theory, labour supply and unemployment duration

Hans G. Bloemen

*Center for Economic Research, Tilburg University, P.O. BOX 90153, 5000 LE Tilburg,  
Netherlands*

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### Abstract

This paper presents a structural model of sequential job search, in which the individual decision makers incorporate labour supply in the job acceptance decision. The model satisfies the reservation wage property. Given the level of the offered wage rate, individuals can choose the number of weekly working hours optimally, by maximizing utility subject to the budget constraint. Specific attention is paid to the stochastic specification. The utility function contains an unobserved random component, and the job offer arrival rate contains unobserved heterogeneity. The search model is used to construct a stationary model of unemployment duration. In estimating the model, simulation methods are used to integrate out unobserved heterogeneity. The goodness of fit of the model is examined by analysis of the residuals.

*Key words:* Job search models; Duration models; Labour supply

*JEL classification:* C41; J22; J64

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### 1. Introduction

This paper presents a model in which elements of job search theory and the labour supply literature are combined. A functional form for the models will be specified and the functional form will be estimated structurally. Flinn and Heckman (1982a) present an overview of the estimation of structural job search models. Applications can be found in Narendranathan and Nickell (1985) and Van den Berg (1990a). In order to find closed-form solutions for the model,

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restrictive assumptions have to be made and this is the reason why the estimation of structural job search models has not become as popular as the reduced form duration models, see Flinn and Heckman (1982b) and Kiefer (1988) for an overview.

Nevertheless, there is still room for extension in the specification of structural job search models as compared with the models which have been estimated up till now. Notably, the assumption that the wage rate is the only component in the objective function of the individual decision maker can be relaxed. The standard job search framework is only concerned with the choice of a job on the basis of the wage rate. On the other hand, there exists an extensive literature on labour supply models. In these models the availability of a job is given and the emphasis is on the participation decision and the choice of the number of hours. Until now few attempts have been made to integrate these two types of models. The present paper presents an empirical search model which extends the standard job search framework with elements of labour supply theory.

In the model, individuals can choose the amount of labour supply optimally, given the level of the offered wage rate. It is shown that the model has the reservation wage property. Optimal labour supply enters the calculation of the reservation wage rate, and, consequently, it enters the job acceptance decision.

In this paper, special attention is paid to the stochastic specification. A random preference component is included in the utility function, and the job offer arrival rate includes an unobserved random component. As a result, the reservation wage rate will be random as well, as opposed to earlier work, e.g. Van den Berg (1990a) and Narendranathan and Nickell (1985). Data on unemployment duration and post unemployment job characteristics are used to estimate the parameters of the utility function, the parameters of the job offer distribution and the job offer arrival rate.

In estimating the model parameters by maximum likelihood, unobserved heterogeneity has to be integrated out. This integration procedure can be costly if it has to be performed numerically, which is definitely the case here, because the integrand contains the reservation wage rate which is the solution of a fixed point problem. To save computing time we can make use of simulation estimators. McFadden (1989) introduced a simulation estimator which is consistent for a fixed number of simulation replications in the context of a multinomial choice model. Bloemen and Kapteyn (1993) adapt it to the limited-dependent variables model and apply it to the neo-classical labour supply model. In their application they also use simulation methods to integrate out an unobserved random preference variable. Various simulation estimators in the context of models with unobserved random variables, and the specific problems which arise in this context, will be discussed.

To arrive at an expression for the reservation wage, restrictive assumptions are usually made in the job search model. One of the more restrictive assumptions is stationarity. To examine the goodness of fit of the model, a plot of the

Kaplan–Meier survivor function is made. To make the residual analysis device suitable for models that include unobserved heterogeneity, a simulation method is employed for simulating the residuals. The residual plot enables us to show to what extent the model fits the data and, moreover, it may reveal the direction of possible non-stationarity.

In Section 2 the economic search model is set up. Attention will be paid to the assumptions that have to be made in order to be able to estimate the model. The reservation wage equation will be derived, specifying the strategy followed by the individual. The likelihood function is specified, and a functional form for the utility function is chosen. Section 3 presents simulation methods for integrating out unobserved heterogeneity. Section 4 presents the data and estimation results. The residual analysis is presented in Section 5, and the final section concludes.

## 2. The model

### 2.1. Job search and labour supply

In this section a job search model is presented in which an unemployed individual maximizes the discounted sum of expected future utility flows, under the assumption that he knows the random process according to which job offers arrive. A job offer is modelled as a random draw from a wage distribution. A decision rule for the acceptance of job offers follows from the utility maximization problem. The model has the reservation wage property. The individual first decides whether or not to accept the wage offer, and if he accepts, hours can be chosen optimally by maximizing the utility function subject to the budget constraint.

In order to be able to find closed-form solutions for the model, some possibly restrictive assumptions have to be made. Most of these assumptions are standard assumptions in structural models of job search. They can be found in Mortensen (1986). We extend these standard assumptions by all wing for the presence of labour supply in the utility function. A similar approach of incorporating labour supply in the context of a job search model can be found in Burdett and Mortensen (1978).

The model assumptions are the following:

1. The individual maximizes a discounted sum of future expected utility flows, subject to the budget constraint and the job offer process:

$$\max_{y_t, h_t} E \int_t^{\infty} u(y_s, h_s; \varepsilon) e^{-\rho(s-t)} ds, \quad (1)$$

where  $y_t$  is income in period  $t$ ,  $h_t$  is labour supply in period  $t$ ,  $\rho$  is the discount rate,  $\varepsilon$  is an individual specific, time independent unobserved random taste parameter, known to the individual. The appearance of the expectation sign refers to the uncertainty about the future state, i.e. the uncertain job possibilities and the

associated wage rates. By the inclusion of labour supply in the utility function we deviate from the standard job search framework, in which the utility function contains income only.

2. The income consists of a state-dependent component and a state independent component (non-labour income). If employed, income equals the sum of labour income and non-labour income:

$$y_t = wh_t + \mu, \quad (2)$$

where  $w$  is the wage rate and  $\mu$  is nonlabour income. For an individual in the state of unemployment, income equals the sum of the unemployment benefit payments  $b$  and the state independent income:

$$y_t = b + \mu. \quad (3)$$

3. A job offer consists of a wage rate  $w$ . Job offers arrive randomly according to a Poisson process with parameter  $\lambda$  from a distribution function  $F(\cdot; m, \tau)$  with accompanying density function  $f(\cdot; m, \tau)$ , with  $m$  the location parameter and  $\tau$  the scale parameter. The distribution function is known to the individual. The domain of  $F(\cdot; m, \tau)$  is  $(0, \infty)$ .

4. The model is stationary, i.e. the job offer arrival rate  $\lambda$ , the unemployment benefit level  $b$ , the wage distribution  $F(w; m, \tau)$  and the non-labour income  $\mu$  are independent of both calendar time and elapsed duration.

5. Once the unemployed has accepted a job it will be kept forever.

6. The utility function has the properties:

$$\frac{\partial u}{\partial y} > 0, \quad (4)$$

$$k(h) := u(wh + \mu, h; \varepsilon), \quad (5)$$

$$k''(h) < 0 \quad \text{for all } h > 0. \quad (6)$$

Under these assumptions it can be derived that there exists a unique reservation wage. All wage offers above the reservation wage rate are acceptable, whereas those below will be rejected.

Assumption 4 is an assumption which we need to arrive at a closed-form solution. Without this assumption the reservation wage rate can only be defined implicitly in terms of a nonlinear differential equation, which can only be solved if not too complicated assumptions for the process of exogenous variables are specified. Van den Berg (1990b) relaxes the stationarity assumptions by introducing general forms of nonstationarity. His empirical application, however, remains restricted to nonstationarity in the benefit level  $b$ , in which, after the unemployment spell has reached a specified length, a discrete jump takes place whereafter it remains constant. In the duration model literature, in which reduced form models are estimated, it turns out to be difficult to distinguish empirically between

negative duration dependence and unobserved heterogeneity, i.e. once unobserved heterogeneity is introduced, the presence or absence of duration dependence is hard to establish. The inclusion of random preferences and randomness in the job offer arrival rate allows us to account for the presence of unobserved heterogeneity. In Section 5, a plot is made of the residuals, based on the estimated parameter values. The shape of the residual plot may reveal information about the direction of possible deviations from non-stationarity.

Assumption 5 is made to be able to find a closed-form expression for the value of a job, which then can be compared with the value of search. This assumption is restrictive if a job is taken on for a short period only, but for someone with a fixed job it is not unreasonable to assume that the individual acts *as if* he will hold the job forever. The plausibility of this assumption also depends on the cost that is related to moving to another job. This cost may arise due to institutional constraints, imposed by the employer or by law, psychic cost, etc. The higher the cost of turnover is, the more plausible Assumption 5 will be. There will be no on-the-job search if cost of turnover is high. The general form of the reservation wage equation remains valid if it is assumed that there is a constant layoff rate (see e.g. Flinn and Heckman, 1982a). That is, we can replace Assumption 5 by Assumption 5':

5'. There exists a constant layoff rate  $\sigma$ . Unless data on transitions from employment into unemployment are used, the layoff rate  $\sigma$  and the rate of time preference  $\rho$  cannot be identified separately. This means that we have to be careful in the interpretation of the estimate of the rate of time preference  $\rho$ . If Assumption 5 is made, whereas there is a nonzero layoff rate, the estimate of  $\rho$  overestimates the rate of time preference. For all the remaining parameters in the model, the estimation results will not change if Assumption 5 is replaced by Assumption 5'.

The final assumption, on the form of the utility function, is made to ensure that a higher wage level is always preferred to a lower one, and to ensure that, once a wage level is chosen, within period utility will reach a maximum if it is maximized with respect to labour supply, subject to the linear budget constraint. Note, that no restrictions are placed on the sign of the derivative of the utility function with respect to labour supply. This is done because results of Narendranathan and Nickell (1985) and Van den Berg (1990c) indicate that individuals might value unemployment, i.e. total leisure, lower than having a job. Their results induce us not to restrict our utility function by the requirement that it is decreasing in the number of working hours everywhere. This implies that the reservation wage rate may become negative in which case any job offer is acceptable to the individual.

The reservation wage  $\xi(\psi)$  is implicitly defined by the following equation, the derivation of which can be found in Appendix A:

$$v(\xi(\psi), \mu; \varepsilon) = u(b + \mu, 0; \varepsilon) + \frac{\lambda}{\rho} \int_{\xi(\psi)}^{\infty} [v(w, \mu; \varepsilon) - v(\xi(\psi), \mu; \varepsilon)] dF(w; m, \tau) \quad (7)$$

in which  $\psi$  refers to unobserved random components, including random preferences  $\varepsilon$  and possible heterogeneity in the arrival rate.  $v(w, \mu; \varepsilon)$  is the indirect utility function which is the result of substituting the labour supply function into the direct utility function. The indirect utility function is well-defined whenever optimal labour supply is positive, i.e. whenever the wage rate  $w$  exceeds the virtual wage rate  $w_0(\varepsilon)$ , which is defined by

$$v(w_0(\varepsilon), \mu; \varepsilon) = u(\mu, 0; \varepsilon). \quad (8)$$

It can easily be shown that the reservation wage rate  $\xi(\psi)$  defined by (7) always exceeds the virtual wage rate  $w_0(\varepsilon)$ . This ensures that labour supply is positive whenever  $w > \xi(\psi)$ .

If Assumption 5 is replaced by Assumption 5',  $\rho$  is replaced by  $\rho + \sigma$  in (7), see e.g. Flinn and Heckman (1982a).

An alternative for the assumption that hours can be chosen optimally, is the assumption that wages and hours are offered as a package. Then it can be shown that there exists a reservation utility level instead of a reservation wage.<sup>1</sup> A special case of the latter model is a model with a single hours choice, restricted at 40 h for example. Then the reservation utility level reduces to a reservation wage, which can be obtained by replacing the indirect utility function in (7) by the direct utility function, evaluated at 40 h.

## 2.2. A structural model of unemployment duration

The search model in the previous subsection is used to construct a structural model of unemployment duration and after unemployment spell job characteristics.

The likelihood contribution for an individual with a completed spell of unemployment and observed after spell job characteristics, is derived for the case of a flow sample. Likelihood contributions for right-hand censored observations, unobserved job characteristics, and a stock sampling scheme, can be derived from this contribution straightforwardly.

The stationarity assumption in Section 2.1 implies that the distribution of completed spells of unemployment, conditional on  $\psi$ , is exponential with escape rate  $\theta(\psi)$ , which loosely speaking equals the probability of getting an acceptable job offer:

$$\theta(\psi) = \lambda \bar{F}(\xi(\psi)) \quad (9)$$

in which  $\bar{F}(\cdot) = 1 - F(\cdot)$ , with  $F(\cdot)$  the wage offer distribution function.

The optimal after spell labour supply  $h^*$  for an individual with after spell wage rate  $w$  and non-labour income  $\mu$  is given by the labour supply function

$$h^* = \bar{h}(w, \mu; \psi). \quad (10)$$

<sup>1</sup>An example of this type of model can be found in Bloemen (1994).

Recall, that optimal labour supply is always positive for wage rates exceeding the reservation wage rate. If we assume that observed labour supply  $h$  is measured with a multiplicative measurement error  $\exp(v)$ ,  $-\infty < v < \infty$ , the density of observed labour supply, conditional on the wage rate and the random taste parameter, can be derived from the density of measurement error  $v$ , by making the transformation

$$h = \bar{h}(w, \mu; \psi) \exp(v). \quad (11)$$

Let the resulting density function be denoted by  $r(h|w, \psi)$ . If an individual is observed to be working at a wage  $w$  and hours  $h$ , the observed wage rate must exceed the reservation wage rate  $\xi(\psi)$ , which implies that the density of observed wages, conditional on  $\psi$ , is truncated. The likelihood contribution of individuals who accepted a job and whose after spell wages and hours are observed and are equal to  $w$  and  $h$ , respectively, and whose unemployment duration equals  $t$ , conditional on  $\psi$ , is given by

$$I(\eta|\psi) = \theta(\psi) \exp\{-\theta(\psi)t\} r(h|w, \psi) \frac{f(w)}{T(\psi)},$$

$$0 < t < \infty, \quad h > 0, \quad w > T(\psi) \quad (12)$$

$$= 0 \text{ otherwise,} \quad (13)$$

where  $\eta$  is a vector containing all model parameters, and  $T(\psi)$  is the truncation probability which is defined by:

$$T(\psi) = \bar{F}(\xi(\psi)) \quad \text{if } \xi(\psi) > 0 \quad (14)$$

$$= 1 \quad \text{if } \xi(\psi) \leq 0. \quad (15)$$

To remove the conditioning on  $\psi$  we have to integrate over all values of  $\psi$  for which  $w > \xi(\psi)$ . Let the marginal density function of  $\psi$  be given by  $g(\psi, \alpha)$ . The unconditional likelihood contribution becomes

$$I(\eta) = \int_{I_w} \theta(\psi) \exp\{-\theta(\psi)t\} r(h|w, \psi) \frac{f(w)}{T(\psi)} g(\psi, \alpha) d\psi,$$

$$t > 0, \quad h > 0, \quad w > 0, \quad (16)$$

where

$$I_w = \{\psi | \xi(\psi) < w\}. \quad (17)$$

The likelihood contribution for right-hand censored observations can be obtained from (16) by integrating out hours and wages, and integrating duration  $t$  over the range  $(0, M)$ , in which  $M$  is the observed, censored, duration. For individuals with a completed spell of unemployment, but whose wage rate



and number of working hours are not observed, the likelihood contribution can be obtained from (15) by integrating out hours and wages.

The likelihood contribution in (16) applies for a flow sample. In case of a stock sample, which is constructed by sampling from the stock of the unemployed, we have to correct for selectivity bias. This can be done by conditioning on backward recurrence times, see e.g. Ridder (1984). The density of duration, conditional on backward recurrence times, can be derived directly from (16). The implicit assumption on the inflow rate which is made in following this procedure is that the inflow rate into the state of unemployment does not depend on the unobserved random variable  $\psi$ .

### 2.3. Specification

In this subsection a specific functional form for the direct utility function is chosen, from which the labour supply function and the indirect utility function can be derived. We use the direct utility function of Hausman (1980).

$$u(y, h; \varepsilon) = -\ln(\gamma - \beta h) - \frac{\beta(h - X\delta - \varepsilon - \beta y)}{\gamma - \beta h}, \quad \beta < 0, \gamma > 0, \quad (18)$$

where  $\varepsilon$  is assumed to be normally distributed with mean zero and variance  $\sigma_\varepsilon^2$  and  $X$  is a vector of individual characteristics. Note, that utility is increasing in income as required by Assumption 6. It can easily be verified that the second condition of Assumption 6 is also satisfied.

Maximizing utility subject to the income equation (2) yields a linear labour supply function in which the disturbance term equals the unobserved random taste parameter:

$$\bar{h}(w, \mu; \psi) = \beta\mu + \gamma w + X\delta + \varepsilon, \quad (19)$$

The virtual wage rate  $w_0(\varepsilon)$ , for which optimal labour supply is exactly equal to zero, defined in (8), is given by

$$w_0(\varepsilon) = -\frac{\beta\mu + X\delta + \varepsilon}{\gamma}. \quad (20)$$

Note, that there are no positivity constraints on this value. Inserting the labour supply function (19) into the direct utility function (18) yields the expression for the indirect utility function:

$$v(w, \mu; \varepsilon) = -\ln(\gamma - \beta X\delta - \beta\varepsilon - \beta\gamma w - \beta^2\mu) - \beta w, \quad (21)$$

which is well defined for all  $w \geq w_0(\varepsilon)$ , and therefore for all  $w \geq \bar{\xi}(\psi)$ .

The extension to a labour supply function that is quadratic in the wage rate is straightforward. Then the utility function in (18) is replaced by the Hausman and Ruud (1984) specification.

The wage offer distribution is taken to be log-normal with log-mean  $\zeta'x$  and log-variance  $\tau$ , respectively, where  $x$  is a vector of individual characteristics. Measurement error  $v$  is assumed to be normally distributed with mean zero and variance  $\sigma_v^2$ . The error  $v$  allows for difference between optimal labour supply, generated by the labour supply function (19), and observed labour supply. The job offer arrival rate can also be made dependent on a vector of characteristics  $z$  by specifying

$$\lambda = \exp(\kappa'z + q), \quad (22)$$

where  $q$  is a normally distributed random variable with mean zero and variance  $\sigma_q^2$ . Consequently, the vector of unobserved components  $\psi$  becomes  $\psi = (\varepsilon, q)'$  and the distribution parameter  $\alpha$  in (16) is  $\alpha = (\sigma_\varepsilon, \sigma_q)'$ . Characteristics which may influence the arrival rate of job offers are individual characteristics like age and sector of education, as well as characteristics of the environment of the individual, like the geographical situation.

### 3. Simulation estimators for models with unobserved heterogeneity

In this section two simulation estimators for models with unobserved random variables which need to be integrated out are discussed. The first method simulates the vector of scores of the log-likelihood function in such a way that the resulting simulated score vector has expectation zero at the true parameter vector. A drawback of the method is that random variables need to be drawn from the distribution of the unobserved random variable, conditional on the observed random variable. The second method, to be considered below, is called smooth simulated maximum likelihood estimation (SSML) as described by Börsch-Supan and Hajivassiliou (1993).

The likelihood function presented in the previous section is of the type in which an unobserved random variable is integrated out. Let the unknown random variable be denoted by  $u$  with marginal density  $\phi(u)$ , where  $u$  may be a vector, containing various sources of randomness, like random preferences and unobserved heterogeneity in arrival rates. Let the observed random variable, or vector of random variables, be denoted by  $y$ , with density function conditional on  $u$  given by  $f(y|u; \eta)$ , in which  $\eta$  is the vector of parameters. The log-likelihood function of a sample of  $N$  i.i.d. observations is

$$L(\eta; y_1, \dots, y_N) = \frac{1}{N} \sum_{i=1}^N \ln \left( \int_{-\infty}^{\infty} f(y_i|u; \eta) \phi(u) du \right), \quad (23)$$

where  $\phi(\cdot)$  does not depend on the parameters of interest, possibly after transformation. It is a well known property of the log-likelihood function that the expectation of the vector of scores equals zero at the true parameter vector  $\eta_0$ .

It is this property that is exploited in the derivation of moment conditions. The vector of scores is

$$\frac{\partial L(\eta|y_1, \dots, y_N)}{\partial \eta} = \frac{1}{N} \sum_{i=1}^N \frac{\int_{-\infty}^{\infty} \partial f(y_i|u; \eta) / \partial \eta \phi(u) du}{\int_{-\infty}^{\infty} f(y_i|u; \eta) \phi(u) du}. \quad (24)$$

The problem is that the integral appears in both the numerator and the denominator and if we want to exploit the possibility of keeping the number of drawings  $R$ , used in the simulation, small, the simulator has to enter the moment conditions linearly. see, e.g. Gouriéroux and Montfort (1989). A simulator can be constructed in the following way. Draw  $R$  i.i.d. random numbers  $u_{ir}$  from  $f(\cdot|y_i)$ , the density of  $u$  conditional on the observed value  $y_i$ , for every individual  $i$ ,  $i = 1, \dots, N$ , and use the moment functions  $S(\eta; y, u)$ :

$$S(\eta; y, u) = \frac{1}{N} \frac{1}{R} \sum_{i=1}^N \sum_{r=1}^R \frac{\partial f(y_i|u_{ir}; \eta) / \partial \eta}{f(y_i|u_{ir}; \eta)}. \quad (25)$$

To show that  $S(\cdot)$  has the same expectation properties as the true vector of scores, the expectation with respect to the drawings  $u_{ir}$  can be calculated, while conditioning on the  $y_i, i = 1, \dots, N$ , i.e. using the conditional distribution of  $u_{ir}$  given  $y_i$ :

$$f(u_{ir}|y_i) = \frac{f(y_i|u_{ir}; \eta) \phi(u_{ir})}{\int_{-\infty}^{\infty} f(y_i|\tilde{u}; \eta) \phi(\tilde{u}) d\tilde{u}}. \quad (26)$$

Taking expectations yields the true vector of scores (24). As a consequence, at the true parameter vector  $\eta_0$  the simulated moments conditions have expectations zero.

The problem is that, in general, there is no rule of drawing random numbers from the conditional density (26) without having to evaluate the integral in the denominator of (26), which we do want to avoid by using simulation methods. In practice the only thing one can do is to approximate the procedure of drawing random numbers from (26) by using an approximate distribution. The most straightforward choice of approximation is to draw the random numbers from the marginal distribution  $\phi(\cdot)$  of  $u$ . At the same time this is also the most naive way of approximating the drawing of random numbers from the conditional distribution because any information about the observed values  $y_i$  is ignored. We now comment on the bias which is introduced if the moments (25) are used with draws  $u_{ir}$  from the marginal density  $\phi(\cdot)$ . Note that the bias, i.e. the difference between the expectation of (25) at the true parameter vector and zero, could be avoided by introducing a weight factor  $w(u_{ir})$ , like in importance sampling, which is the ratio of the true density function (26) and the density one actually draws from, in this case the marginal density function  $\phi(\cdot)$ .

$$w(u_{ir}) = \frac{f(u_{ir}|y_i)}{\phi(u_{ir})} = \frac{f(y_i|u_{ir}; \eta)}{\int_{-\infty}^{\infty} f(y_i|\tilde{u}; \eta) \phi(\tilde{u}) d\tilde{u}} \quad (27)$$

which again contains the integral to be simulated. So in order to investigate the bias, we have to look at the consequences of ignoring the weight function. Note that the expectation of the weight function equals one by construction. As a consequence, the size of the bias is closely related to the variance of the weight function. The smaller the variance of the weight function, the smaller the bias in the simulated score equations will be, see e.g. Kloek and Van Dijk (1978). (Note that the weight function is identically equal to one if the  $u_{ir}$  are drawn from the conditional density function  $f(\cdot|y_i)$ .)

If the bounds of the integral are a function of the parameters, an additional complication arises, i.e. we have to take the derivatives with respect to the bounds. Suppose we have a likelihood contribution given by

$$L_i = \ln \left[ \int_{a_i(\eta)}^{b_i(\eta)} f(y_i|u; \eta) \phi(u) du \right]. \quad (28)$$

The derivatives are:

$$\begin{aligned} & \frac{[\partial b_i(\eta)/\partial \eta f(y_i|b_i(\eta); \eta) \phi(b_i(\eta)) - \partial a_i(\eta)/\partial \eta f(y_i|a_i(\eta); \eta) \phi(a_i(\eta))]}{\int_{a_i(\eta)}^{b_i(\eta)} f(y_i|u; \eta) \phi(u) du} \\ & + \frac{\int_{a_i(\eta)}^{b_i(\eta)} \partial f(y_i|u; \eta)/\partial \eta \phi(u) du}{\int_{a_i(\eta)}^{b_i(\eta)} f(y_i|u; \eta) \phi(u) du}. \end{aligned} \quad (29)$$

An unbiased simulator for the score vector now is

$$\begin{aligned} & \frac{1}{NR} \sum_{i=1}^N \sum_{r=1}^R \frac{[\partial b_i(\eta)/\partial \eta f(y_i|b_i(\eta); \eta) \phi(b_i(\eta)) - \partial a_i(\eta)/\partial \eta f(y_i|a_i(\eta); \eta) \phi(a_i(\eta))]}{f(y_i|u_{ir}; \eta) \int_{a_i(\eta)}^{b_i(\eta)} \phi(u) du} \\ & + S(\eta; y, u) \end{aligned} \quad (30)$$

with  $u_{ir}$  drawings from  $f(u_{ir}|y_i, a_i(\eta) < u_{ir} < b_i(\eta))$ .

A second simulation estimator can be obtained by simulating the likelihood function rather than looking at the vector of scores. The integrals which appear in (23) are replaced by a simulator. Unbiased simulators for the integrals can be obtained by drawing random numbers  $u_{ir}$  from the marginal density  $\phi(\cdot)$  and calculating

$$\frac{1}{R} \sum_{r=1}^R f(y_i|u_{ir}; \eta). \quad (31)$$

Because of the logarithmic transformation in (23) the resulting estimator will be inconsistent for a fixed and small value of  $R$ , which actually is the reason why we considered simulation estimators based on the vector of scores before. However, if the simulator is a smooth function of the parameters, the method functions satisfactorily even for smaller values of  $R$ . This is shown in Börsch-Supan and

Hajivassiliou (1993), who call this method smooth simulated maximum likelihood estimation, where they have added the word “smooth” in order to stress the fact that one needs a simulator which is a smooth function of the parameters, rather than a frequency type of simulator, in order to get a satisfactory performance.

#### 4. Data and estimation results

##### 4.1. *The data*

The data come from the Socio-economic Panel (SEP), which is a survey, carried out in Netherlands every six months in April and October by the Central Bureau of Statistics (CBS). In the survey, the participating individuals are asked to report their occupational status for every month of the past six months. The data used are those of the waves of October 1985 through the October 1987 wave. Selected are male individuals who reported to be unemployed in any month during the observation period, which means that the sample partly has a stock character and partly a flow character. It has been determined how many months they remained unemployed, and for the individuals in the stock sample, the backward recurrence times are recorded. For most of the individuals whose unemployment spell ended during the observation period, data on their after spell hourly wage rate and the weekly number of hours are available. The sample consists of 516 individuals. The number of complete unemployment spells is 272. The remaining 244 spells are truncated. For 211 of the 272 individuals whose unemployment spells are completed the after spell job characteristics, i.e. the hourly wage rate and the weekly number of working hours, are observed.

In Table 1 some sample statistics are given. In the survey there are five levels of education where level 1 is the lowest and level 5 the highest. The mode of the level of education is 2. We have divided the Netherlands into four regions. Region 1 is the most strongly industrialized part of the Netherlands which includes the larger cities. Region 4 is the least industrialized part of the Netherlands with a relatively low population density and a sizeable agricultural sector. Region 3 is the south of Netherlands containing some large companies and agricultural industry. In region 2 (the east) there is a mix of industry and agriculture. Apart from having information about the *level* of education we have information available about the type or *sector* of education. Sector 1 is the technical sector which includes chemistry, physics, mathematics and biology, sector 2 includes the economic and administrative studies, sector 3 is non-vocational training education and sector 4 includes services.

##### 4.2. *Estimation results*

Table 2 presents estimates obtained with simulated maximum likelihood, using  $R = 10$  drawings. The estimates of the utility parameters, imply that optimal

Table 1  
Sample statistics

Variable	Mean	Standard deviation
Working hours (h/week)	39.0	9.1
After tax hourly wage rate (guilders/h)	10.1	4.5
Benefits (guilders/week)	289.6	108.2
Non-labour income (guilders/week)	80.4	188.8
Age	31.2	11.8
Family size (persons)	3.2	1.7
Education level	Mode 2	
Dutch nationality	93.6%	
Region 1 (industrialized west)	31.6%	
Region 2 (east)	29.3%	
Region 3 (south)	26.6%	
Region 4 (agricultural)	12.6%	
Sector of education 1 (technical)	30.4%	
Sector of education 2 (economic/administrative)	8.7%	
Sector of education 3 (no specialization)	48.8%	
Sector of education 4 (services)	12.0%	

labour supply is not very sensitive with respect to non-labour income and the wage rate. The wage elasticity of labour supply is 0.12. Family size has a positive effect on the optimal amount of labour supply. Age has a negative effect on the job offer arrival rate, i.e. the older one is, the fewer job offers can be expected. The nationality dummy, which is one for those who do not have the Dutch nationality, is negative, indicating that having the Dutch nationality has a positive influence on the job offer arrival rate. The coefficients of the regional dummies have a positive sign, which means a higher job offer arrival rate for people living outside the agricultural region 4. Only the dummies for region 1 and region 3 are significant. Living in the industrial west is associated with a higher job offer arrival rate than living in the rest of the country. Only the sectoral dummy for sector 3, which includes those individuals who only had non-vocational training, is significant. The highest wages are offered to the individuals with the highest level of education.

The parameter estimate of  $\rho$  is 0.0049. As unemployment duration is measured in months, this means that the monthly discount rate is 0.49%, or equivalently, the discount rate is 5.9% per year.

As noted before, estimates obtained by Narendranathan and Nickell (1985) and Van den Berg (1990c) showed that unemployment, i.e. total leisure, was assigned a lower utility value than employment. Their reservation wage equation is of the form

$$u(\varphi + \mu) - \omega u(b + \mu) = \frac{\lambda}{\rho} \int_{\varphi}^{\infty} (u(x) - u(\varphi)) dF(x) \quad (32)$$

Table 2

Estimation results with simulated maximum likelihood,  $R = 10$ 

The utility parameters		
$\beta$	$-5.05 \times 10^{-4}$	0.0046
$\gamma$	0.45	0.21
$\delta_1$ , Constant	32.06	3.65
$\delta_2$ , Log family size	2.04	2.09
$\sigma_e$	1.64	0.03
The job offer arrival rate parameters		
$\kappa_1$ , Constant	0.46	6.61
$\kappa_2$ , Log age	-0.13	3.94
$\kappa_3$ , Nationality	-0.99	0.39
$\kappa_4$ , Region 1	0.72	0.27
$\kappa_5$ , Region 2	0.33	0.27
$\kappa_6$ , Region 3	0.47	0.28
$\kappa_7$ , Sector 1	-0.075	0.26
$\kappa_8$ , Sector 2	-0.31	0.35
$\kappa_9$ , Sector 3	-0.59	0.26
$\kappa_{10}$ , Square of log age	-0.19	0.58
$\sigma_{\kappa}$	1.09	0.27
Rate of time preference		
$\rho$	0.0049	0.87
The wage distribution		
$\zeta_1$ , Constant	-12.13	3.26
$\zeta_2$ , Log age	8.09	1.77
$\zeta_3$ , Square of log age	-1.11	0.28
$\zeta_4$ , educ1	-0.32	0.08
$\zeta_5$ , educ2	-0.35	0.08
$\zeta_6$ , educ3	-0.25	0.08
$\tau$	0.30	0.0037
Distr. of measurement error		
$\sigma_{\epsilon}$	0.32	0.08

in which  $\varphi$  is the reservation wage income and  $\omega$  is the utility parameter whose estimates have been found to be less than one. Note that the right-hand side of (32) is always positive, from which we can derive that

$$u(\varphi + \mu) > \omega u(b + \mu). \quad (33)$$

Now  $\omega \geq 1$  implies that the reservation wage  $\varphi$  is always higher than the benefit income  $b$ . However, if  $\omega < 1$ , the reservation wage is allowed to be lower than the benefit rate. A value of the reservation wage which is actually below the

benefit rate indicates that unemployment must have a lower utility value than employment. Our specification has enough flexibility for the labour income at the reservation wage rate to be below the income from benefits, but it cannot be checked directly, by looking at a single parameter, whether unemployment is valued lower than employment. Therefore, we conduct a simulation by drawing random preferences  $\varepsilon$  and unobserved heterogeneity  $q$  for each individual, calculating the reservation wage rate  $\xi(\psi)$ , computing the optimal labour income at the reservation wage rate if  $\xi(\psi) > 0$ , i.e.  $\xi(\psi)h(\xi(\psi), \mu; \psi)$  and comparing it with the benefit income  $b$ . This procedure was repeated 100 times and the result is that for 87% of the sample the reservation income is lower than the benefit income. This result is consistent with the findings of Narendranathan and Nickell (1985) and Van den Berg (1990c), who have estimated a value of  $\omega$  in (32) that is smaller than 1, i.e. unemployment lowers utility.

## 5. Residual analysis and simulated frequencies

In order to be able to say something about the performance of the model we will do some residual analysis. An overview of residual analysis of duration models can be found in Lancaster (1990) and Cox and Oakes (1984). There are two points according to which the standard residual analysis in duration models cannot be applied directly to our model. The first point is that this analysis is usually performed in the context of a flow sample, whereas we have a sample which consists of a flow subsample as well as a stock subsample. Therefore, in the analysis of the residuals we restrict ourselves to the individuals who are in the flow subsample. The second point is that residual analysis is usually performed on models which do not contain a random unobserved heterogeneity component. This problem is solved in the following way. The marginal density of observed duration  $t$  is given by  $f(t)$ :

$$f(t) = \int_{-\infty}^{\infty} \theta(\tilde{\psi}) \exp\{-\theta(\tilde{\psi})t\} g(\tilde{\psi}, \alpha) d\tilde{\psi}, \quad 0 < t < \infty. \quad (34)$$

The density of the unobserved heterogeneity variable  $\psi$ , conditional on observed duration  $t$  is given by

$$g(\psi|t) = \frac{\theta(\psi) \exp\{-\theta(\psi)t\} g(\psi, \alpha)}{\int_{-\infty}^{\infty} \theta(\tilde{\psi}) \exp\{-\theta(\tilde{\psi})t\} g(\tilde{\psi}, \alpha) d\tilde{\psi}}, \quad -\infty < \psi < \infty. \quad (35)$$

Now draw a random number  $\psi_r$  from  $g(\cdot|t)$ . This can be done by using the inversion method, see e.g. Devroye (1986). Then the pair  $(t, \psi_r)$  can be seen as



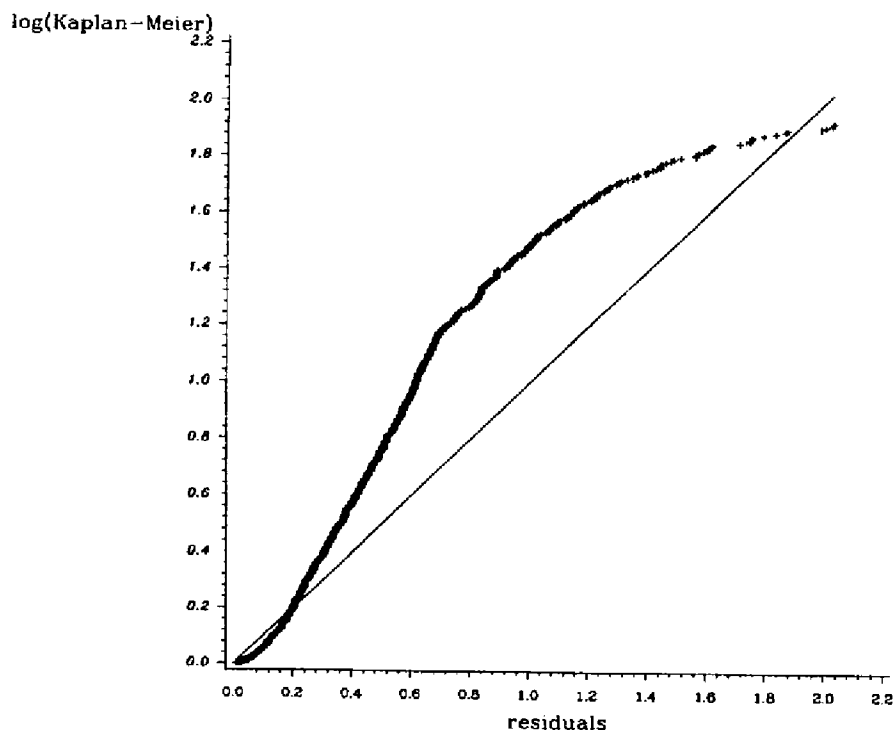


Fig. 1. log(Kaplan-Meier) vs. residuals.

a joint draw from

$$f(t)g(\psi|t) = \theta(\psi) \exp\{-\theta(\psi)t\}g(\psi, x), \quad 0 < t < \infty, \quad -\infty < \psi < \infty. \quad (36)$$

Consider the following transformation:

$$\begin{aligned} \vartheta &= \theta(\psi)t, \\ \psi &= \psi. \end{aligned} \quad (37)$$

The jacobian of the transformation is  $1/\theta(\psi)$ . The joint density of  $\vartheta$  and  $\psi$  becomes

$$\begin{aligned} f\left(\frac{\vartheta}{\theta(\psi)}\right)g\left(\psi \middle| \frac{\vartheta}{\theta(\psi)}\right) \\ = \exp\{-\vartheta\}g(\psi, \alpha), \quad 0 < \vartheta < \infty, \quad -\infty < \psi < \infty \end{aligned} \quad (38)$$

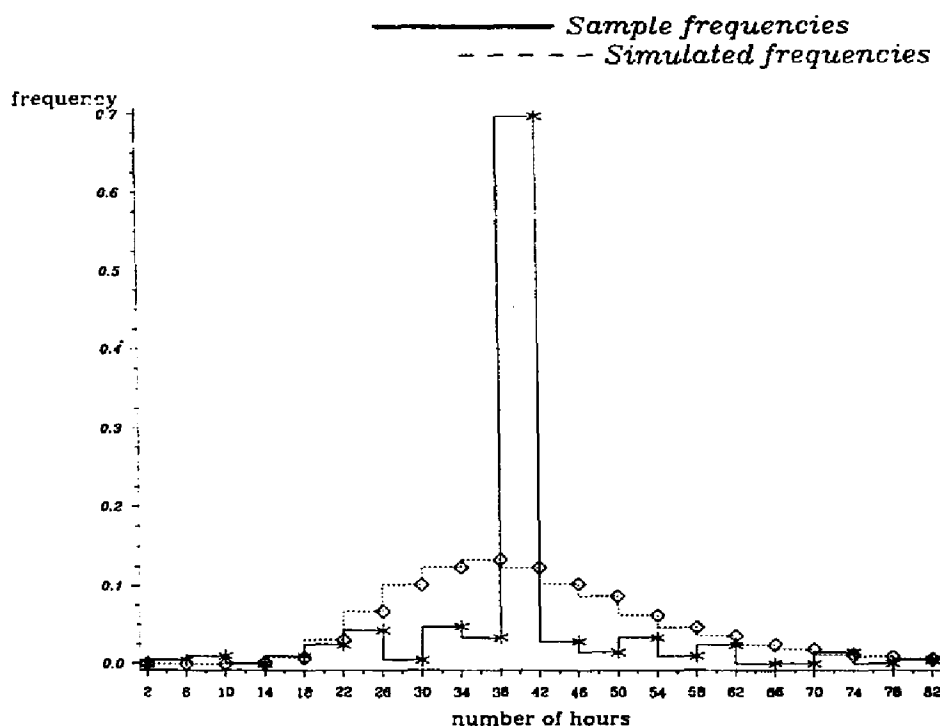


Fig. 2. Frequencies of working hours per week.

which implies that  $\vartheta$  is exponentially distributed with parameter 1, which enables us to apply the standard residual analysis. Summarizing, the procedure is:

- Draw  $\psi_r$  from  $g(\cdot|t)$ , where  $g(\cdot|t)$  is evaluated in the parameter estimates and  $t$  is observed duration.
- Calculate  $\hat{\vartheta}_r = \hat{\theta}(\psi_r)t = \theta(\psi_r, \hat{\eta})t$  in which  $\hat{\theta}(\psi_r)$  is the hazard rate evaluated in the parameter estimate  $\hat{\eta}$ .  $\hat{\vartheta}_r$  is the simulated residual. For every individual, several residuals can be calculated by drawing several random numbers.
- Calculate the Kaplan–Meier estimate of the survivor function in the residuals  $\hat{\vartheta}_{ir}, i = 1, \dots, N, r = 1, \dots, R$ , in which the subindex  $i$  is over individuals and the subindex  $r$  is over different draws from the conditional density.

The residuals can be plotted against minus the logarithm of the Kaplan–Meier estimate. If the parametric model is correctly specified, the plot would be approximately a 45° line. Various forms of misspecification of the hazard rate can cause the residual plot to deviate from the 45° line. Lancaster (1990) shows that omitting an unobserved heterogeneity factor in the hazard rate leads to underdispersion, i.e. the plot will be below the 45° line. Particularly interesting for our

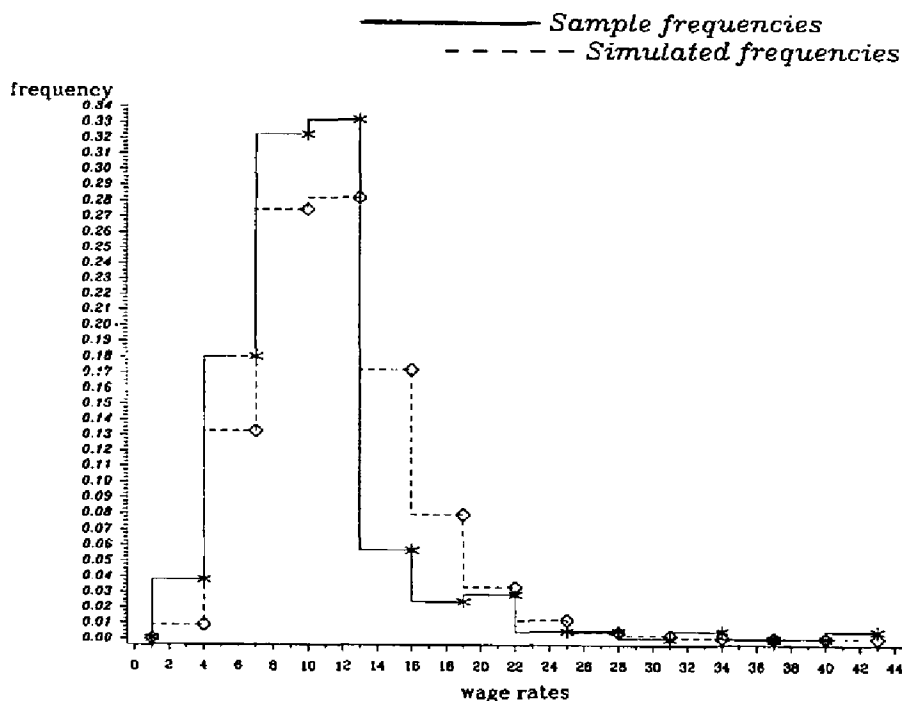


Fig. 3. Frequencies of after tax wage rates.

application are the deviations which are caused by wrongly assuming that the hazard rate is constant, i.e. the stationarity assumption. Ridder (1987) shows that when the data exhibit positive duration dependence, whereas a constant hazard model is estimated, the residual plot will be above the 45° line. In the case of negative duration dependence of the hazard rate, the reverse holds.

For every individual, five residuals have been simulated. Fig. 1 shows the plot of minus the logarithm of the Kaplan-Meier estimate versus the residuals. The plot is above the 45° line, indicating that there could be positive duration dependence.

Fig. 2 shows the frequency distribution of simulated hours, conditional on observed wages, and the sample frequencies of observed hours. It is clear that the model does not manage to generate the peak level around 40 h a week.

Fig. 3 shows the frequencies of simulated wages and the sample frequencies of wages.

## 6. Conclusions

A model of job search has been presented. In the model, individuals can determine their labour supply optimally, given the wage rate. The model has the reservation wage property. Optimal labour supply is accounted for in the calculation of the reservation wage.

The stochastic specification of the model includes random preferences, as well as unobserved heterogeneity in the arrival rate, which induces the reservation wage to be random as well. In the estimation of the model, the unobserved randomness has to be integrated out. In this paper, simulation techniques are employed in the integration procedure. The use of simulation techniques is particularly useful in handling multidimensional integrals.

Parameter estimates of the job offer arrival rate show that age has a negative effect on the job offer arrival rate. People who live in the industrialized western part of the Netherlands have a larger probability of getting a job offer. Individuals with non-vocational training have a lower chance of getting a job offer than other individuals. For individuals who have the Dutch nationality the job offer arrival rate is larger than for others.

The reservation income for every individual has been simulated and compared with the benefit income. A large percentage of the simulated reservation incomes were below the benefit incomes. This is consistent with results of Van den Berg (1990c) who finds evidence in favour of “disutility of unemployment”.

Residual analysis shows that the hazard rate is possibly positive duration dependent, i.e. the longer someone is unemployed, the larger the escape rate is. Positive duration dependence of the hazard rate can be caused by negative duration dependence of the reservation wage rate, possibly due to negative duration dependence of benefit payments or positive business cycle movements.

The simulated frequencies of part time jobs, full time jobs and high-level jobs reveal that the poor performance of the model is mainly caused by the assumption made about labour supply.

## Appendix A. Derivation of reservation wage equation

Let  $V(\psi)$  denote the value of search of an individual with unobserved heterogeneity vector  $\psi$ , which includes  $\varepsilon$ . Due to the stationarity assumption (Assumption 4 in Section 2.1),  $V(\psi)$  is independent of time. At time  $t$ , the individual is not working, is looking for a job and receives weekly non-labour income  $\mu$  and weekly benefits  $b$ , which are time independent due to Assumption 4 in Section 2.1. In a short time interval of length  $\Delta t$  the utility flow derived from  $\mu$  and  $b$  equals

$$\int_t^{t+\Delta t} u(b + \mu, 0; \varepsilon) e^{-\rho(s-t)} ds = \frac{u(b + \mu, 0; \varepsilon)}{\rho} (1 - e^{-\rho \Delta t}). \quad (\text{A.1})$$

In the time interval of length  $\Delta t$  there is a probability of  $e^{-\lambda \Delta t} \lambda \Delta t + o(\Delta t)$  of receiving a job offer, consisting of a wage rate  $\tilde{w}$ . The value of the job, denoted by  $W(\tilde{w}; \psi)$  will be compared with the value of continuing searching, which is  $V(\psi)$ . The job offer  $\tilde{w}$  will be accepted if  $W(\tilde{w}; \psi)$  exceeds  $V(\psi)$ . Due to the Assumption 5 in Section 2.1, the value will remain  $W(\tilde{w}; \psi)$  once a job  $\tilde{w}$  is accepted. With probability  $1 - e^{-\lambda \Delta t} \lambda \Delta t + o(\Delta t)$  the individual does not get a job offer in the time interval of length  $\Delta t$ , in which case the value remains at  $V(\psi)$ . Summarizing, the value  $V(\psi)$  is

$$\begin{aligned} V(\psi) = & u(b + \mu, 0; \varepsilon)(1 - e^{-\rho \Delta t})/\rho \\ & + e^{-\rho \Delta t} \{ (1 - e^{-\lambda \Delta t} \lambda \Delta t) V(\psi) \\ & + e^{-\lambda \Delta t} \lambda \Delta t E_{\tilde{w}} \max[V(\psi), W(\tilde{w}; \psi)] \} + o(\Delta t). \end{aligned} \quad (\text{A.2})$$

Rearranging terms yields and letting  $\Delta t \rightarrow 0$  we obtain

$$\rho V(\psi) = u(b + \mu, 0; \varepsilon) + \lambda E_{\tilde{w}} \max[0, W(\tilde{w}; \psi) - V(\psi)]. \quad (\text{A.3})$$

Using the Assumptions 1, 2, 4–6, the value of the job with wage rate  $\tilde{w}$  can be obtained by solving the maximization problem

$$\max_{c, h} \frac{u(y, h; \varepsilon)}{\rho}, \quad \text{subject to } y = \tilde{w}h + \mu. \quad (\text{A.4})$$

The solution for  $h$  is  $h^* = \bar{h}(\tilde{w}, \mu; \varepsilon)$  which is the neo-classical labour supply function. Inserting the solution for  $y$  and  $h$  in the direct utility function yields the indirect utility function which is  $v(\tilde{w}, \mu; \varepsilon)$  and, therefore,

$$W(\tilde{w}; \psi) = \frac{v(\tilde{w}, \mu; \varepsilon)}{\rho}. \quad (\text{A.5})$$

A job offer  $\tilde{w}$  will be accepted if  $v(\tilde{w}, \mu; \varepsilon) > \rho V(\psi)$ . If the reverse holds, it will be rejected. As the indirect utility function is increasing in the wage rate, there exists a unique reservation wage rate  $\xi(\psi)$  such that all wages above it are acceptable and those below will be rejected. The value of search is equal to the value of accepting a job if

$$\rho V(\psi) = v(\xi(\psi), \mu; \varepsilon). \quad (\text{A.6})$$

Inserting (A.5) and (A.6) in (A.3) and using the distributional assumptions on  $\tilde{w}$  results in the reservation wage equation:

$$\begin{aligned} v(\xi(\psi), \mu; \varepsilon) = & u(b + \mu, 0; \varepsilon) + \frac{\lambda}{\rho} \int_{\xi(\psi)}^{\infty} [v(\tilde{w}, \mu; \varepsilon) \\ & - v(\xi(\psi), \mu; \varepsilon)] dF(\tilde{w}; m, \tau). \end{aligned} \quad (\text{A.7})$$

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